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ADIABATIC OSCILLATIONS OF THE ONE-DIMENSIONAL SGEMP BOUNDARY LAYER

Mission Research Corporation

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Adiabatic oscillations of the one-dimensional SGEMP boundary layer about equilibrium are described in a fluid approximation. The solution for mono-energetic emission normal to the surface is found and compared with a computer solution. Solutions for photoelectron emission with a cosine distribution for a monoenergetic and a linear times exponential energy distribution are also found. It appears that these latter solutions are not physically meaningful. —> next page

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20. ABSTRACT (Continued)

cont. If physically meaningful modes of oscillation do exist, for other emission
configurations, they are probably stable.

SUMMARY

I am indebted to Dr. Conrad Longmire and Dr. Neal Carron for writing a very useful and readable report. Thanks go to Dr. Carron for setting up the parameters for the computer solution (although it was originally used for a different purpose) presented in Appendix II. Thanks also go to Dr. Longmire for helpful comments after reading the first draft of this report. I am indebted to Ms. Colleen Hannigan for her efforts in typing this report.

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SECTION 1

INTRODUCTION

When high-intensity X-rays strike a conducting surface in
vacuum a negatively-charged layer forms just above the surface. This
layer—called the space-charge-boundary layer—is formed by the photo-
electrons ejected from the surface. The properties of the boundary layer,
in particular the modes of oscillation, can be important in the study of
SGEMP.

Recent experiments using low-energy X rays to form such a boundary
layer may not be wholly interpretable in terms of purely static (time
independent) boundary layer theory. The unambiguous interpretation of
future experiments, with low-energy photons, will depend upon an understand-
ing of the dynamic properties of the boundary layer. This paper attempts
to contribute to that understanding by investigating small oscillation of
the boundary layer about static equilibrium.

To properly analyze the dynamic properties of the one-dimensional
boundary layer requires the simultaneous solution of the Vlasov equation
and the Poisson equation. Since this is a non-linear system of equations
its complete time-dependent solution is most easily made by means of
computer codes. A computer analysis of this problem has been made by
Carron[1]. The analytic investigation of small oscillations about equilibrium
will be made by means of the linearized Vlasov equation and by utilizing an
analog of the adiabatic condition for an ideal gas. This latter condition

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is equivalent to solving the linearized second order Vlasov moment equation (energy equation).

The Vlasov equation used to solve the boundary layer system contains only one species, namely electrons. The system is therefore not a plasma in the literal sense of the word. Although some of the terminology of plasma physics is convenient for a description of the boundary layer, concepts such as the frequency of plasma oscillations should not be accepted uncritically. The definition of a plasma requires that the plasma system is neutral outside a length called a Debye length. Plasma theory describes the plasma system only for dimensions greater than a Debye length. Exactly the opposite is true for the boundary layer. There exists a length, called the boundary layer thickness (sometimes also called a Debye length) which basically defines the dimension beyond which the system is neutral. For the boundary layer all the physics happens within this length not outside of it. One of the results of this investigation will be the determination of the frequencies of adiabatic oscillation of the boundary system for known static solutions. A general way to approximate the lowest mode of oscillation for a general static solution will also be described.

In Section 2 of this report the wave equation, describing oscillations of the boundary layer, for a general static solution is derived. In Section 3 the condition for adiabatic oscillations is formulated and the wave equation of Section 2 is particularized to the case of adiabatic oscillations. Boundary conditions are also discussed in this section. One of interesting results that follows from the properties of the new wave equation is that all fluid oscillations—if they exist at all—of any boundary layer, are stable.

In Section 4 we solve for the modes of oscillation for three known static solutions[2,3]. These static solutions are for electrons emitted

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either normal to the surface or for electrons emitted with a cosine distribution with respect to the normal to the surface. One of these static solutions describes normal emission with a monoenergetic energy distribution. This is the only static solution of the three which allows physically meaningful solutions. This result seems to correspond with the findings of computer code investigations of the one-dimensional boundary layer. Oscillations have been observed with monoenergetic, normal emission code simulations but the oscillations do not seem to occur with other types of emission.

In Section 5 the results of the reported investigation are summarized and speculated upon. In Appendix I we demonstrate that the means by which we obtained the adiabatic condition for the boundary layer yields the usual adiabatic condition in the case of an ideal gas. Appendix II contains a comparison between the oscillations observed in a computer simulation of monoenergetic normal emission with that of the analytic result of Section 4. The lowest mode of the analytic result is equal to the measured frequency, within the accuracy of the measurement.
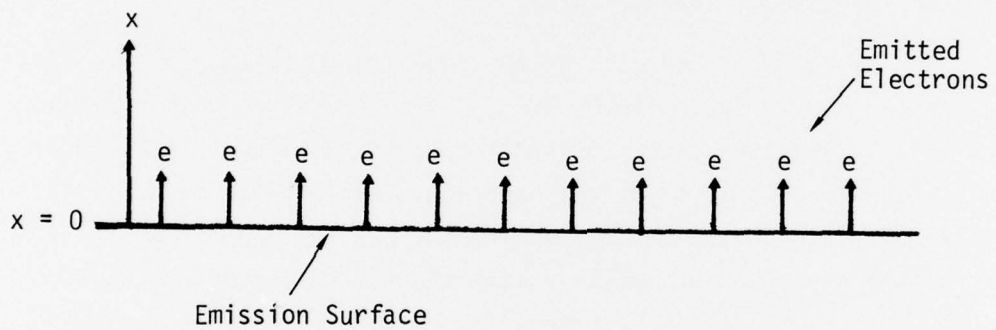
Figure 1. Geometry of the problem.

SECTION 2

GENERAL EQUATION OF OSCILLATIONS

In this section the wave equation which describes oscillations about equilibrium will be derived. We take the point of view that the perturbations of physical interest manifest themselves in terms of an observable fluid velocity $\partial\xi/\partial t$ (an observable charged current flow, for example). ξ can be interpreted as a displacement of a particle of fluid from its equilibrium position. We look for those equations that describe the displacement ξ. We begin by taking the usual zero order and first order velocity moments of the linearized Vlasov equation. Combining these two equations we will obtain the equation we seek.

The Vlasov equation in one dimension is

$$\frac{\partial f}{\partial t} + v \frac{\partial f}{\partial x} + a \frac{\partial f}{\partial v} = 0 , \tag{1}$$

where $f(x,v,t)$ is the distribution function for electrons, x is the variable of position, v is the velocity variable, t is the time and a is the acceleration. The acceleration is defined by

$$a = - e/m \frac{\partial \phi}{\partial x} , \tag{2}$$

where the electrostatic potential ϕ is defined through the Poisson equation

$$\frac{\partial^2 \phi}{\partial x^2} = - 4\pi (e/m)\rho , \tag{3}$$

and the mass density is defined by

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$$\rho(x,t) \equiv m \int f dv \ . \tag{4}$$

We now linearize Equation 1. If f_0 represents a static solution of Equations 1 through 3 and δf represents a perturbation about equilibrium, then using

$$f = f_0 + \delta f \ , \tag{5}$$

in Equations 1 to 3 the equations describing the perturbation becomes

$$\frac{\partial(\delta f)}{\partial t} + v \frac{\partial(\delta f)}{\partial x} + a_0 \frac{\partial(\delta f)}{\partial v} + \delta a \frac{\partial(f_0)}{\partial v} = 0 \ , \tag{6}$$

$$\frac{\partial^2(\delta \phi)}{\partial x^2} = - 4\pi \ e/m \ \delta\rho \ , \tag{7}$$

and

$$\delta a = - \frac{e}{m} \frac{\partial(\delta \phi)}{\partial x} \ . \tag{8}$$

Here we have used the static equilibrium equation

$$v \frac{\partial f_0}{\partial x} + a_0 \frac{\partial f_0}{\partial v} = 0 \ , \tag{9}$$

and the notation that a subscript zero corresponds to a quantity defined by means of the static distribution function f_0, a δ before a quantity is defined by means of the perturbation δf. For example, the mass densities ρ_0 and $\delta\rho$ are defined by inserting f_0 and δf respectively in Equation 4 for f:

$$\rho_0 = m \int f_0 dv \ , \tag{10}$$

and

$$\delta\rho = m \int \delta f dv \ . \tag{11}$$

Taking the zero order and first order moments of Equation 6 while using the definitions

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$$\rho_0 \frac{\partial \xi}{\partial t} \equiv m \int v \delta f dv \ , \tag{12}$$

and

$$P \equiv m \int v^2 f dv \ , \tag{13}$$

where $\partial \xi / \partial t$ is the perturbed fluid velocity and P is the pressure, we find that

$$\frac{\partial(\delta\rho)}{\partial t} + \frac{\partial^2(\rho_0 \xi)}{\partial t \partial x} = 0 \ , \tag{14}$$

and

$$\rho_0 \frac{\partial^2 \xi}{\partial t^2} + \frac{\partial \delta P}{\partial x} - a_0 \delta\rho - \delta a \rho_0 = 0 \ . \tag{15}$$

Substituting Equation 2 (with ϕ replaced by ϕ_0) and Equation 8 into Equation 15 we find

$$\rho_0 \frac{\partial^2 \xi}{\partial t^2} + \frac{\partial}{\partial x} \delta P + \frac{e}{m} \left(\frac{\partial \phi_0}{\partial x} \delta\rho + \frac{\partial \delta\phi}{\partial x} \rho_0 \right) = 0 \ . \tag{16}$$

Equations 14 and 16 are defined totally in terms of fluid concepts. From Equation 14 the expression for the conservation of mass, we see that

$$\delta\rho = - \frac{\partial}{\partial x} (\rho_0 \xi) \ . \tag{17}$$

Upon substituting (17) into (7) we have

$$\frac{\partial \delta\phi}{\partial x} = - 4\pi \ e/m \rho_0 \xi \ . \tag{18}$$

Substituting Equations 17 and 18 into 16, using Equation 7 and the equation of static equilibrium

$$\frac{\partial P_0}{\partial x} = - \frac{e}{m} \rho \frac{\partial \phi_0}{\partial x} \ , \tag{19}$$

we find that

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$$\rho_0 \frac{\partial^2 \xi}{\partial t^2} + \frac{\partial}{\partial x} \left(\delta P + \frac{\partial P_0 \xi}{\partial x} \right) = 0 \ . \tag{20}$$

Equation 20 is the general differential equation we seek which describes oscillations about static solutions. In the next section we will relate δP to ξ for a particular type of motion.

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SECTION 3

ADIABATIC OSCILLATION CONDITIONS

To find an expression for δP in Equation 20 we formulate a condition which maintains the energy in any given volume of fluid constant throughout the perturbation. In Appendix I we demonstrate that this condition is analogous to adiabatic motion in an ideal gas. The condition in the circumstances which describe the boundary layer, will also be called the adiabatic condition.

If the energy density of the electron gas at x is $\epsilon(x)$ then the total energy W is

$$W = \int_0^d \epsilon(x)dx \; . \tag{21}$$

Taking the point of view that ξ is the displacement of the electron gas at the point x it can be shown[4] that the change in energy δw due to this displacement is

$$\delta W = \int_0^d \left(\delta\epsilon + \frac{\partial(\epsilon_0\xi)}{\partial x} \right)dx \; . \tag{22}$$

If we wish δW to equal zero for every element of volume we must have

$$\delta\epsilon = -\frac{\partial}{\partial x}(\epsilon_0\xi) \; . \tag{23}$$

Equation 23 is the condition we need but to use it we must construct the energy density of the electron gas.

The time derivative of Equation 23 is the second order moment equation of the linearized Vlasov equation and defines the heat flow as

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$\varepsilon_0 \; \partial\xi/\partial t$. Instead of obtaining Equation 23 by means of Equations 21 and 22 we could, alternatively, have assumed that the heat flow was given by $\varepsilon_0 \; \partial\xi/\partial t$ and solved the linearized energy equation.

The energy density ε_0 is composed of the electric field energy and the kinetic energy of the electrons. In general the energy density $\varepsilon(x)$ is given by

$$\varepsilon(x) = \frac{1}{8\pi} \left(\frac{\partial\phi}{\partial x}\right)^2 + \frac{1}{2} P \; , \tag{24}$$

and so

$$\delta\varepsilon = \frac{1}{4\pi} \frac{\partial\phi_0}{\partial x} \frac{\partial\delta\phi}{\partial x} + \frac{1}{2} \delta P \; . \tag{25}$$

Substituting Equations 18 and 19 into (25) we find

$$\delta\varepsilon = - \frac{\partial P}{\partial x} \xi + \frac{1}{2} \delta P \; . \tag{26}$$

From Equations 3 and 19 one can show that

$$\frac{\partial P_0}{\partial x} = + \frac{1}{8\pi} \frac{\partial}{\partial x} \left(\frac{\partial\phi}{\partial x}\right)^2 \; , \tag{27}$$

or that

$$P_0 = \frac{1}{8\pi} \left(\frac{\partial\phi_0}{\partial x}\right)^2 \; , \tag{28}$$

if both P_0 and $\partial\phi_0/\partial x$ vanish at the outer boundary. Substituting Equation 28 into Equation 25 we have

$$\varepsilon_0 = 3/2 \; P_0 \; . \tag{29}$$

Equation 29 states that the total energy density in the one-dimensional boundary layer, for normal emission is three times the kinetic energy. If the electrons were not emitted normally the expression for ε_0 would not be Equation 29. For example if electrons were emitted with a $\cos\theta$ distribution

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with respect to the normal to the surface the fraction multiplying P_0 in Equation 29 would be 5/2 instead of 3/2 (see Reference 2 pgs. 21 through 23). Substituting Equations 26 and 29 into Equation 23 we arrive at the condition we seek, namely

$$\delta P + P \frac{\partial \xi}{\partial x} = - 3P_0 \frac{\partial \xi}{\partial x} . \tag{30}$$

For $\cos\theta$ emission 5/3 would replace the 3 on the right-hand side of Equation 30. (This is so because a 3/2 would replace the 1/2 multiplying δP on the right-hand side of Equation 25 for $\cos\theta$ emission.) Substituting Equation 30 into Equation 20 we arrive at the wave equation for oscillations

$$\rho_0 \frac{\partial^2 \xi}{\partial t^2} - \gamma \frac{\partial}{\partial x} (P_0 \frac{\partial \xi}{\partial x}) = 0 , \tag{31}$$

where

$$\begin{aligned} \gamma &= 3 \qquad \text{normal emission} \\ \gamma &= 5/3 \quad \cos\theta \text{ emission} . \end{aligned} \tag{32}$$

Equation 31 is analogous to the equation describing the velocity of sound in a gas. γ is analogous to the ratio of specific heats. Since for a one-dimensional ideal gas one would expect the ratio of specific heat C_p/C_v to be

$$\frac{C_p}{C_v} = \frac{3/2 \ k}{1/2 \ k} = 3 , \tag{33}$$

where C_p is the specific heat at constant pressure, C_v is the specific heat at constant volume and k is Boltzman's constant. For an ideal gas with 3 degrees of freedom the ratio of specific heats would be 5/3. Equation 33 is a result of the Boltzman distribution function. Equation 32 is not, although the results are similar. We will now discuss boundary conditions and a general method of finding the lowest mode of Equation 31. If we assume that

$$\xi = \xi_0(x)e^{\omega t} , \tag{34}$$

then substituting Equation 34 into Equation 31 we find that

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$$\rho_0 \omega^2 \xi_0 - \gamma \frac{\partial}{\partial x} \left(P_0 \frac{\partial \xi_0}{\partial x} \right) = 0 \ . \tag{35}$$

The spatial boundary conditions for Equation 35 can be found by an examination of Equation 35. For $x < 0$, ξ_0 is equal to zero. Equation 35 implies that $\partial \xi_0 / \partial x$ is continuous at $x = 0$ so that (this argument is analogous to the "pill box" argument used in describing the boundary condition at the surface of a conductor):

$$\left. \frac{\partial \xi_0}{\partial x} \right|_{x=0} = 0 \ , \tag{36}$$

is the boundary condition at the inner surface. Equation 36 expresses the fact that the fluid is incompressible at the $x = 0$ boundary and is also consistent with our intuition that the perturbation should be, at least, a local extremum at the boundary. Other boundary conditions are conceivable and they could charge the nature of our conclusions about the frequencies of oscillations (the form of the particular solutions discussed in Section 4 would not change if the boundary conditions were changed, however). It is the opinion of this author that Equation 36 is the most "natural" condition for this system.

For $x > d$, ξ_0 is equal to zero also. Continuity of ξ_0 at the outer boundary would require $\xi_0(d)$ to equal zero. To make the perturbations physically meaningful we require ξ_0 to be finite everywhere including the position of the outer boundary.

We can investigate the roots (ω^2) of Equation 35 by doing a few manipulations. Multiplying Equation 35 by ξ_0, integrating over x and using the fact that $P_0(d)$ vanishes, while also using Equation 36 we see that

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$$\omega^2 = -\gamma \frac{\int_0^d P_0 \left(\frac{\partial \xi}{\partial x}\right)^2 dx}{\int_0^d \rho_0 (\xi)^2 dx} \quad . \tag{37}$$

Equation 37 is the basis for an approximation of ω^2. It also shows us that ω is either imaginary or zero since both the integrals on the right-hand side are positive definite. That is the adiabatic oscillations are never unstable! Equation 37 forms the basis of a variational principle: any function $f(x)$ which satisfied the boundary conditions on ξ_0 can be substituted into Equation 37; the resulting integrations make the right-hand side of Equation 37 greater or equal to the lowest mode of the system. The variational principle arises because Equation 35 together with the boundary conditions are a self-adjoint system.

SECTION 4

MODES OF OSCILLATION OF KNOWN SOLUTIONS

In this section we find the modes of oscillation of three known static solutions. The first is a monoenergetic energy distribution emitted normal to the surface. We need both P_0 and ρ_0 as functions of x. From Reference 2 (we use the variable x instead of z) page 25 we have, after using Equation 28

$$P_0 = a(1 + \frac{x}{\ell})^{3/2} , \tag{38}$$

$$\rho_0 = b(1 + \frac{x}{\ell})^{-3/2} , \tag{39}$$

where

$$\ell = - d , \tag{40}$$

$$a = 2 m r_0 v_1 , \tag{41}$$

and

$$b = \frac{2 m r_0}{v_1} . \tag{42}$$

r_0 is number of electrons emitted /cm^2/sec and v_1 is the emission velocity. Substituting Equations 38 and 39 into Equation 35 while letting

$$y = (1 + x/\ell) , \tag{43}$$

and

$$\beta^2 \equiv - \frac{\omega^2 b}{\gamma a} \ell^2 , \tag{44}$$

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we have

$$\frac{\partial}{\partial y}\left(y^{2/3} \frac{\partial}{\partial y} \xi_0\right) + \beta y^{-2/3} \ . \tag{45}$$

By means of a simple transformation

$$y = (z' \frac{1}{3})^3 \ . \tag{46}$$

Equation 46 can be transformed to

$$\frac{\partial^2 \xi_0}{\partial^2 z'} + \beta^2 \xi_0 = 0 \ . \tag{47}$$

The solutions are then

$$\xi_0 \propto \cos\left[3\beta(1 - x/d)^{1/3}\right] \ , \tag{48}$$

and

$$\xi_0 \propto \sin\left[3\beta(1 - x/d)^{1/3}\right] \ . \tag{49}$$

In order to satisfy continuity of ξ_0 at d we choose Equation 49 as our solution. To satisfy Equation 36 for $\beta \neq 0$ we must have

$$\beta = \frac{2n+1}{6} \pi \quad n = 0, 1 \ldots \tag{50}$$

or

$$\omega_n = \pm i \left(\frac{\gamma a}{bd^2}\right)^{1/2} \frac{2n+1}{6} \pi \ , \tag{51}$$

or from Equations 41, 42 and 32

$$\omega_n = \pm i \frac{v_1}{d} \frac{2n+1}{2\sqrt{3}} \pi \quad n = 0, 1, 2 \ldots \tag{52}$$

The longest period is 3.46 multiplied by the time it takes on electron with velocity v_1 to go twice the boundary distance d. If we used the solution of Equation 48, ξ_0 would be discontinuous at $x = d$ and the frequency would be:

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$$\omega_m = \pm\, i\, \frac{v_1}{d}\, \frac{m\pi}{\sqrt{3}} \qquad m = 0,1,2 \ldots \tag{53}$$

A comparison of the lowest mode of Equation 52 with a computer solution is made in Appendix II.

We now proceed to the next static solution. It can be interpreted as either a constant energy spectrum with normal emission or a monoenergetic energy distribution with a $\cos\theta$ emission spectrum. With either interpretation the results are the same. From Reference 2 pages 26 and 30 we have

$$P_0 = a(1 + x/\ell)^6 \,, \tag{54}$$

$$\rho_0 = b(1 + x/\ell)^2 \,, \tag{55}$$

where

$$\ell = -\, d \,, \tag{56}$$

$$a = m\, r_0\, v_1/3 \,, \tag{57}$$

and

$$b = 4\, r_0/v_1 \,. \tag{58}$$

Here constants r_0 and v_1 have the same meaning they had in the previous example. Using the transformation expressed by Equations 54, 55, 43, and also Equation 44 we find that Equation 36 becomes

$$\frac{\partial}{\partial y}\left(y^6 \frac{\partial}{\partial y} \xi_0 \right) + \beta^2 y^2 \xi_0 = 0 \,. \tag{59}$$

If we let

$$\xi_0 = y^{-3}\mu \,, \tag{60}$$

and

$$y = z^{-1} \,, \tag{61}$$

Equation 59 transforms to

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$$\frac{\partial^2 \mu}{\partial z^2} + \frac{2}{z} \frac{\partial \mu}{\partial z} - \frac{6}{z^2} + \beta^2 \mu = 0 \ . \tag{62}$$

The solutions of Equation 62 are spherical Bessel functions of order 2. If H_2 represents a spherical Hankel function of order 2 then the solutions are

$$\xi_0 \propto y^{-3} H_2(\beta/y) \ . \tag{63}$$

In order for ξ_0 to be finite at $x = d$ ($y = 0$), β must be imaginary. Referring to Equation 44 we see then that ω would be real which contradicts the conclusions drawn from Equation 37. (If we assumed that ω were real we could not satisfy the boundary condition at $x = 0$.) Physically realistic solutions of ξ_0 do not exist for this static solution.

We next consider the static solution which describes normal emission with an exponential distribution or a cosine angular distribution with a linear times exponential energy distribution. From Reference 2, page 31

$$P_0 = a(1 + x/\ell)^{-2} \ , \tag{64}$$

$$\rho_0 = b(1 + x/\ell)^{-2} \ , \tag{65}$$

where

$$\ell = \left(\frac{2w_1 v_1}{8\pi^{3/2} e^2 r_0} \right) , \tag{66}$$

$$a = \pi^{1/2} m r_0 v_1 \ , \tag{67}$$

and

$$b = 2\pi^{1/2} \frac{r_0}{v_1} \ . \tag{68}$$

The constants have the meaning they had previously except that w_1 is the exponentiation energy of the energy distribution (e^{-w/w_1}) and

$$v_1 \equiv \sqrt{2w_1/m} \ .$$

(69)

Using Equations 64, 65, 43 and Equation 44 we find that Equation 35 becomes

$$\frac{\partial}{\partial y}\left(y^{-2}\frac{\partial}{\partial y}\xi_0\right) + \beta^2 y^{-2} = 0 \ .$$

(70)

With the help of the transformation

$$\xi_0 = y^2\mu \ ,$$

(71)

Equation 70 becomes

$$\frac{\partial^2\mu}{\partial y^2} + 2/y\frac{\partial^2\mu}{\partial y} - 2/y^2\mu + \beta^2\mu = 0 \ .$$

(72)

The solutions of Equation 72 are spherical Bessel functions of order 1. If H_1 denotes a Hankel function of order 1 then

$$\xi_0 \propto y^2 H_1(\beta y) \ .$$

(72)

In order for ξ_0 to be finite at $y = \infty$, β must be negative or ω must be positive. By arguments similar to those of the previous section we conclude that a physically meaningful perturbed solution does not exist for this static solution.

SECTION 5

SUMMARY AND SPECULATION

Adiabatic fluid theory has been used successfully in plasma physics to describe Langmuir waves[5]. Here we have evoked an adiabatic fluid theory to describe oscillations of the one-dimensional boundary layer. The results of the analysis (utilizing a "natural" boundary condition for the problem) are that a boundary layer formed from monoenergetic emission of electrons normal to the surface supports plasma-type oscillations. A boundary layer formed by either a monoenergetic energy distribution of electrons emitted with a cosine angular dependence or a linear times exponential energy distribution of electrons emitted with a cosine angular dependence do not support oscillations.

These results, together with the form of the equation (Equation 35) describing the oscillations, suggest that monoenergetic emission of electrons normal to the surface may be one of the only conditions under which oscillations can occur in a one-dimensional boundary layer. This suggestion is supported by the one-dimensional computer simulations done at MRC with the SCALE1D code[1]. If a velocity space analysis were undertaken it would probably show that perturbations of the electron gas are critically damped for distributions other than that of monoenergetic normal emission.

If in fact there are other emission configurations which allow oscillations of the boundary layer and these oscillations are described by an adiabatic fluid, Section 3 demonstrates that the frequencies of oscillations must be real. This means that the static boundary layer is stable with respect to adiabatic perturbations.

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The implications for experiments is that any instability that arises within an experiment with a one-dimensional boundary layer probably comes from an interaction with the experimental system and the layer but not from the electron gas itself. A linear times exponential energy distribution with cosine emission may be particularly relevant to photon experiments. Under these circumstances the electron gas does not support oscillations itself and probably damps oscillations occurring because of an interaction with an external system.

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APPENDIX I

In this appendix we demonstrate that the adiabatic condition for an ideal gas, under constraints similar to that of the SGEMP boundary layer, follows directly from Equation 23. We consider the idealized circumstance of a one-dimensional system of gas held together by self-gravitation rather than electric forces. The energy density ε, including gravitational energy ε is

$$\varepsilon = \frac{1}{8\pi G} \left(\frac{\partial \phi}{\partial x}\right)^2 + U , \tag{I-1}$$

where ϕ is the gravitational potential, U is the internal energy of the gas per unit volume and G is the gravitational constant. If α is the number of degrees of freedom, then assuming a Boltzman distribution

$$U = \alpha/2 \; P . \tag{I-2}$$

Because of the equation of equilibrium

$$P_0 = \frac{1}{8\pi G} \left(\frac{\partial \phi}{\partial x}\right)^2 . \tag{I-3}$$

Substituting Equations I-2 and I-3 into Equation I-1 we see that

$$\varepsilon_0 = \frac{\alpha+2}{2} P_0 . \tag{I-4}$$

Substituting Equation I-4 into Equation 23 we have

$$\delta\varepsilon = - \frac{\alpha}{2} \gamma \frac{\partial}{\partial r} (P_0 \xi) , \tag{I-5}$$

where we have recognized γ as the ratio of specific heats. By using the Poisson equation for the gravitational system one can show, in a way

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analogous to the derivation of Equation 25, that

$$\delta\varepsilon = \frac{\alpha}{2}\delta P + \frac{\partial P_0}{\partial x}\xi \; .$$

(I-6)

Substituting Equation I-6 into I-5 we find that

$$\delta P = -\gamma P_0 \frac{\partial\xi}{\partial x} - \frac{\partial P_0}{\partial x}\xi \; .$$

(I-7)

Equation I-7 is the correct adiabatic relation for one-dimensional perturbations of an ideal gas.

APPENDIX II

In this appendix we compare a computer solution made with the
code[1] SCALE1D for normal monoenergetic emission with the analytical result
of Section 4. We compare the period of the lowest mode derived analytically
with the oscillations observed in the computer simulation. We find the
results agree to within the accuracy of measurement.

The code solution describes the physical situation where electrons
are emitted normally with a very nearly monoenergetic energy distribution.
The emission energy is 1 keV. The time history of the emission spectrum
rises to a maximum in 10 nanoseconds along a linear ramp. It remains
constant for 50 nanoseconds and then drops discontinuously to zero. The
output of the code is in terms of normalized parameters. The relevant
ones are:

Time — 1.010×10^{-9} sec

Length — 1.892 cm

Velocity — 1.873×10^{9} cm/sec

Charge Density — 1.479×10^{-1} ESU/cm^3

Current Density — 9.239×10^{-3} abamps/cm^2

Figure 2 shows the emission velocity distribution. Figure 3 shows the
charge density as a function of distance from the surface at a time large
enough to correspond roughly to equilibrium. For the static problem the
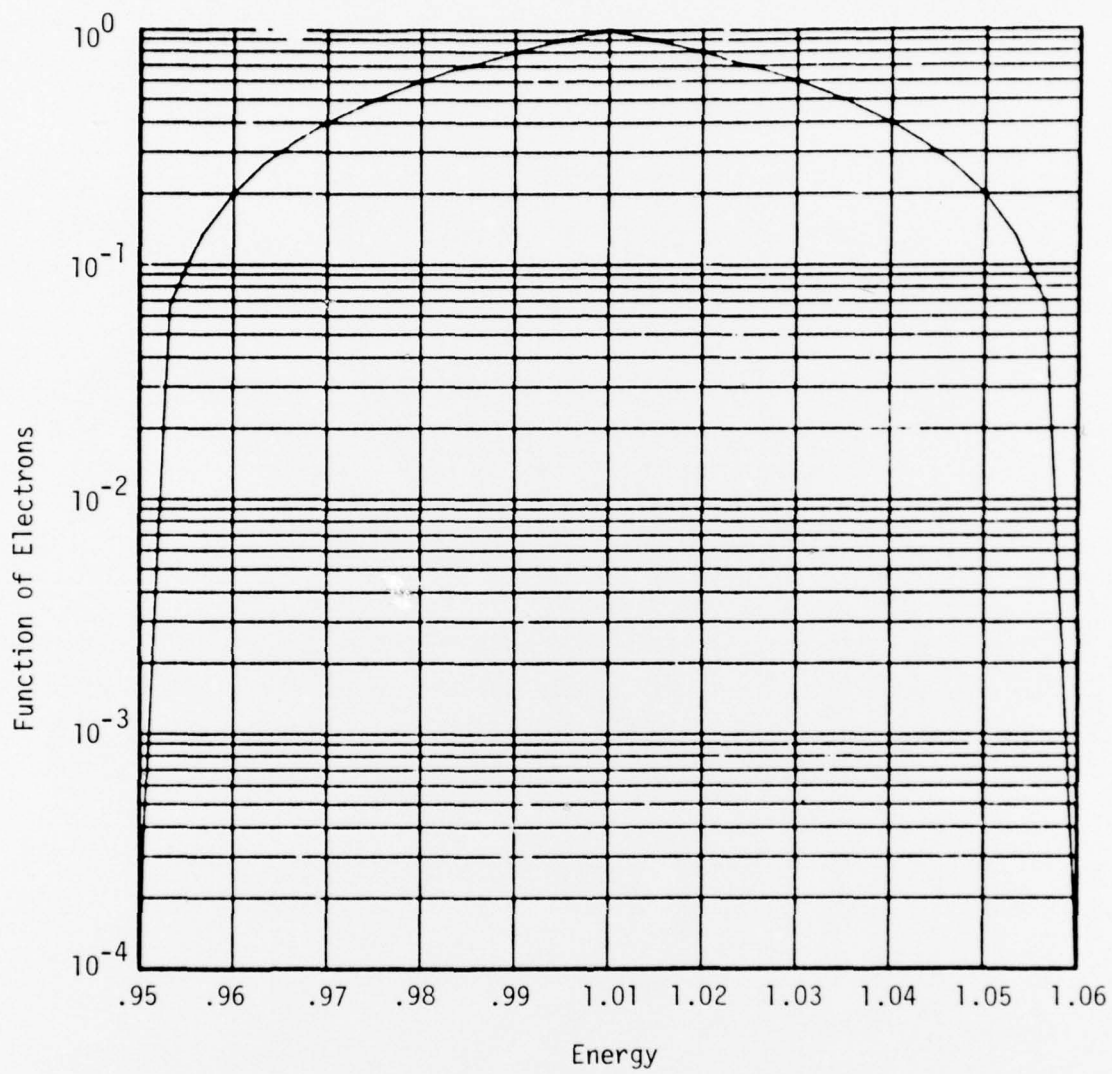charge density (Equation 38) approaches infinity for $x = d$. In the computer
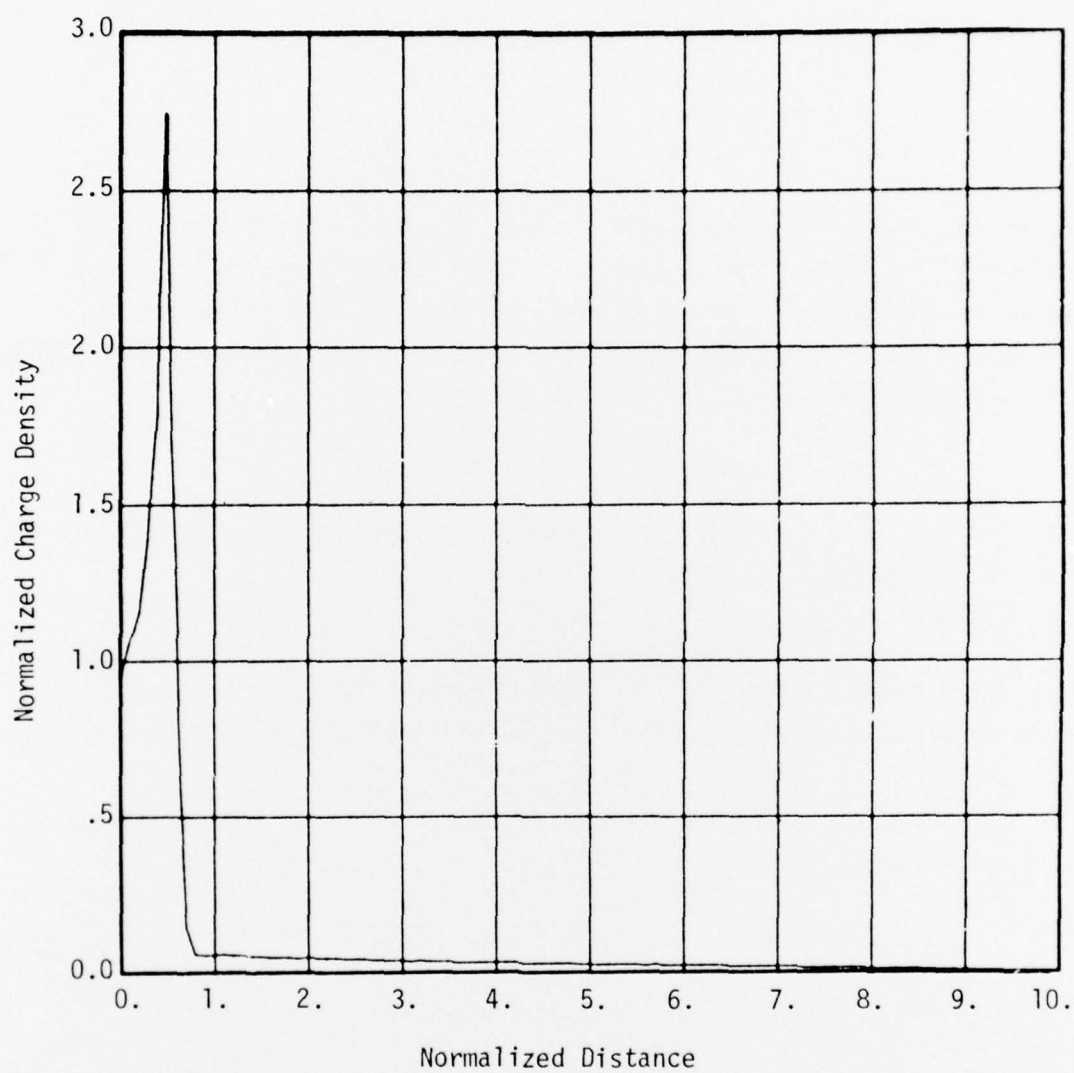
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Figure 2. Emission energy distribution.

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Figure 3. Charge density vs distance from the emitting surface (at a
time 9.4).

solution the charge density is discontinuous at d. From Figure 2 this distance is (.467)(1.892) = .883. From the static solution theory (Reference 2, page 24) the distance d is given in terms of

$$d = 2/3 \sqrt{\frac{w_1 v_1}{8\pi e^2 r_0}} \quad . \tag{II-1}$$

The parameters in the above equation have the same meaning they did in Section 4. w_1 is the emission energy. In the computer problem 1.73×10^{10} electrons are emitted / cm^2 over the total time of 60 ns. To find an average emission current we divide the emitted electrons by a time of 55 ns (10 ns of the time history is a ramp). Thus

$$r_0 = 3.15 \times 10^{17} \ cm^{-2} \ sec^{-1} \quad . \tag{II-2}$$

Substituting this r_0 together with the appropriate values of the other parameters into Equation II-1 we find that

$$d = .855 \ cm \quad . \tag{II-3}$$

This value corresponds quite well with the value taken from Figure 2 and lends credence to the assumption that we are dealing with a static solution.

Figure 3 shows the current density at .568 cm from the emitting surface. From the graphs the oscillations have a period τ_0 of

$$\tau_0 = 3.24 \times 1.01 \times 10^{-9} = 3.26 \times 10^{-9} \ sec \quad . \tag{II-4}$$

The value for the lowest mode (n=0) in Equation 52 is

$$\tau_0 = (3.46)\frac{2d}{v_1} = \frac{(3.46)(2)(.883)}{1.873 \times 10^{-9}} = 3.26 \times 10^{-9} \ sec \quad .$$

If we had used the value of d expressed by Equation II-3, we would have obtained τ_0 equal to 3.16×10^{-9} sec. This number is within .3 percent of the measured value.*

---

* Dr. Longmire has pointed out that the oscillations depicted in Figure 4 are not really the sinusoidal oscillations suggested by the theory. The oscillations of Figure 4 appear to be actually non-linear. A code run designed specifically to describe oscillations about an actual equilibrium solution would be a better check of the theory.
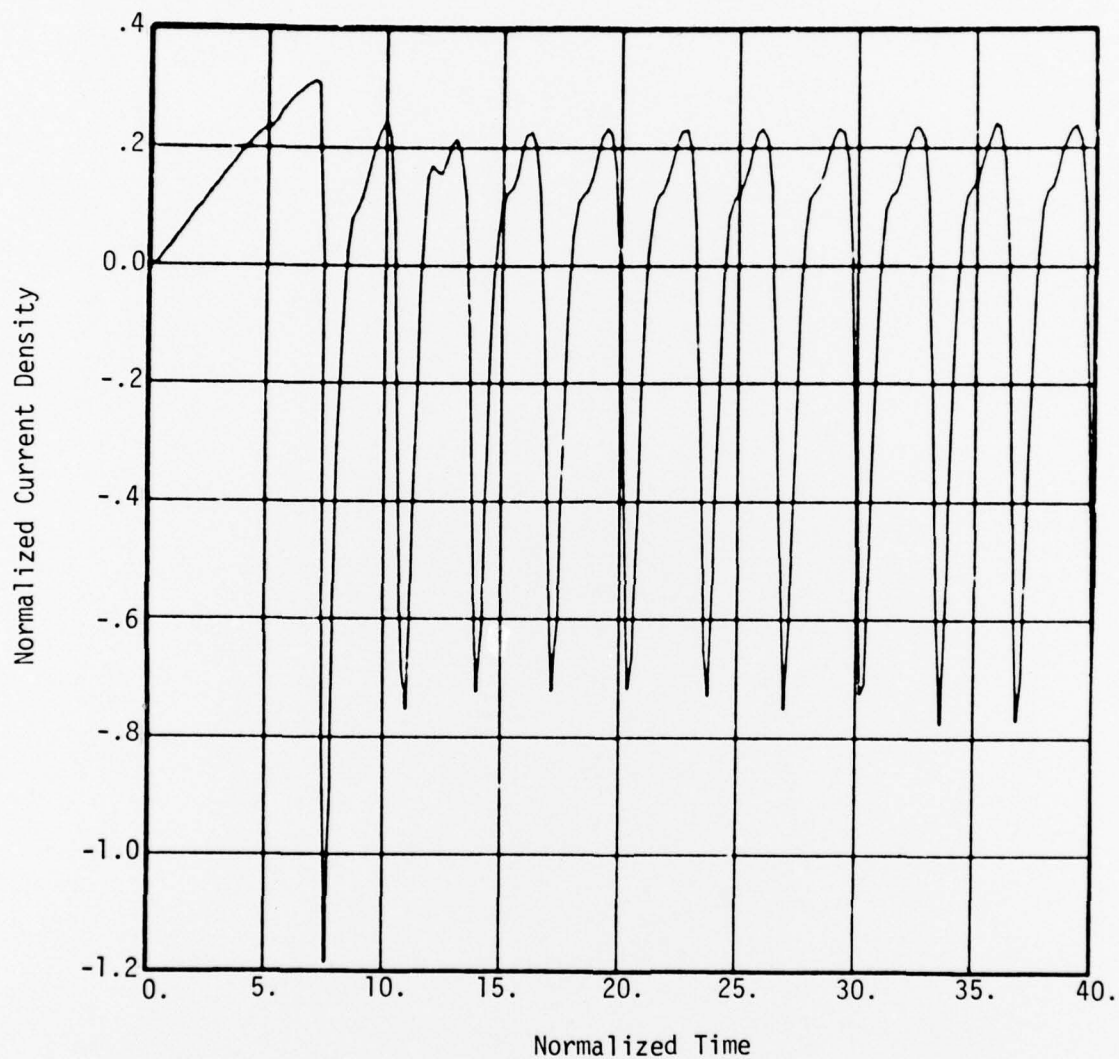
Figure 4.   Current density vs. time (at .3 length units from the emission surface).

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